'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2021

DSE-P2-PHYSICS

The figures in the margin indicate full marks.
All symbols are of usual significance.

Candidates should also ensure that the chosen section in the paper DSE-2 is different from the chosen section in the paper DSE-1.

The question paper contains paper DSE-2A, DSE-2B and DSE-2C. The candidates are required to answer any *one* from *three* sections. Candidates should mention it clearly on the Answer Book.

DSE-2A

NANO-MATERIALS AND APPLICATIONS

Time Allotted: 2 Hours **Full Marks: 40**

GROUP-A

1. Answer any ***five*** questions from the following: $1\times5 = 5$

(a) Which factor causes the properties of nano-materials to differ significantly from other materials? 1

(b) Which nano-materials is used for cutting tools? 1

(c) A carbon monoxide sensor made of zinconia uses which characteristic to detect any charge? 1

(d) If the atomic numbers of zirconium, molybdenum, palladium and tin are 40, 42, 46 and 50 respectively, which will be suitable filter for X-radiation from molybdenum? 1

(e) Define Band gap. 1

(f) What do you mean by nanowires? 1

(g) Define grain boundary of a nanoparticle. 1

(h) What is a quantum-dot laser? 1

GROUP-B

Answer any *three* questions from the following $5\times3 = 15$

2. (a) Define Bragg's law. 2

(b) Find the longest wavelength that can be used to analyse a NaCl crystal of interplanar spacing 0.281 nm between its principal planes in first order. 3

3. (a) Distinguish between direct and indirect band gap. 3
   (b) What is exciton? Explain. 2

4. Discuss in detail application of nanosensor systems. 5

5. Explain in detail why band gap of nano-materials increases with size reduction. 5

6. Discuss in detail different types of ball-milling and their advantages. 5

GROUP-C

Answer any *two* questions from the following $10\times2 = 20$

7. Discuss several bottom up approaches to synthesize nano-materials. 10

8. List out applications of nano-materials and neatly explain them. 10

9. (a) Explain exciton generation and its transport in quantum dots. 6
   (b) What is the difference between SEM and STM? 4

10. (a) Explain Coulomb interactions in a dielectric quantum nanostructure. 4
    (b) Calculate the self energy and charging energy when the quantum dot is embedded in a semi-conductor with large band gap. 3+3

DSE-2B

ADVANCED MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours **Full Marks: 40**

GROUP-A

1. Answer any ***five*** questions from the following: $1\times5 = 5$

   (a) Find the Laplace transform of the signal 1

   $$x(t) = te^{-2|t|}.$$

   (b) Draw the graph of $\theta(t-a)-\theta(t-b)$. θ is defined as step functions a and b are arbitrary constant. 1

   (c) Show that $\vec{a} = 2\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{b} = 6\hat{i}+9\hat{j}+15\hat{k}$ do not form any closed surface. 1

   (d) If A is a $(n \times n)$ antisymmetric matrix, show that $|A| = 0$ when n is an odd integer number. 1

(e) Find the dimension of the subspace of $M_{2\times 2}$ spanned by, 1

$$\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix}, \text{ and } \begin{pmatrix} 2 & -4 \\ -5 & -7 \end{pmatrix}$$

(f) Two directions $\vec{n}$ and $\vec{n}'$ are defined in a spherical coordinate system by the angles θ, α and θ', α respectively. Find the cosine of the angle between them. 1

(g) Write down the basis of a rank-2 tensor in 2-dimension. 1

(h) Calculate δ_{ii} in 3-dimension. 1

GROUP-B

Answer any *three* questions from the following 5×3 = 15

2. Obtain Inverse Laplace Transform of 5

$$\frac{s}{1+s^2+s^4}$$

3. (a) Define a linear functional on a vector space. 2

(b) Consider the vector space $\mathbb{R}[x]$ of all polynomials over the field $\mathbb{R}$ of real numbers. Show that the mapping $f(x) \to \int_0^1 f(x)\,dx;\ f(x) \in \mathbb{R}[x]$ is a linear functional on $\mathbb{R}[x]$. 3

4. (a) Write down the condition on which a subset of a vector space can be called linearly dependent. 2

(b) Check the linear independency of the set, 3
$S = \{(1, 3, -4, 2), (2, 2, -4, 0), (1, -3, 2, -4), (-1, 0, 1, 0)\}$ in $\mathbb{R}^4$.

5. (a) Construct a scalar from the tensor A^{ij}_{kl}. 3

(b) Define metric tensor. 2

6. (a) Find out the basis transformation matrix (S) in 3-D when the Cartesian coordinate is rotated with an angle θ about x-axis. 2

(b) The vector field $\vec{a}$ satisfies $\nabla \cdot \vec{a} = 0$ inside some volume V and $\vec{a} \cdot \hat{n} = 0$ on the boundary surface S. $\hat{n}$ is the unit vector along $\vec{S}$. By considering the divergence theorem applied to $T_{ij} = x_i a_j$, show that $\int_V \vec{a}\, dV = 0$. 3

GROUP-C

Answer any *two* questions from the following 10×2 = 20

7. Solve the initial value problem 10

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0,$$

Where, $y = 2$ at $x = 0$, $\frac{dy}{dx}=-4$ at $x = 0$.

8. (a) What do you mean by the linear 'dimension' of a vector space? 2

(b) Justify whether every subspace of a finite dimensional vector space is finite dimensional or not. 3

(c) Find the dimension of the vector space formed by all (2×2) matrices. 3

(d) Explain with examples whether the dimension of a vector space depends on its field or not. 2

9. Let, $A=\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 3 \\ -1 & -2 & 0 & 2 & 3 \end{bmatrix}$.

(a) Solve $Ax=0$ and characterize the null space through its basis. 3

(b) What is the rank of A? What are the dimensions of the column space, row space and left null space of A? 2

(c) Find the complete solution of $Ax=b$, where $b=\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$. 3

(d) Find the conditions on b_1, b_2, b_3 that ensure $Ax=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ has a solution. 2

10.(a) Show that, in general coordinates, the quantities $\frac{\partial v^i}{\partial u^j}$ do not form the components of a tensor. 3

(b) Prove that δ^i_j is a mixed second rank tensor. 2

(c) A covariant rank-1 tensor has components $xy,\, 2y-z^2,\, xz$ in rectangular coordinates. Find its covariant components in spherical coordinates. 5

DSE-2C

CLASSICAL DYNAMICS

Time Allotted: 2 Hours **Full Marks: 60**

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Prove that a possible Lagrangian for a free particle is, 3

$$L = \dot{q}^2 - q\dot{q}$$

(b) What are the Lagrange's equations for a non-conservative system? 3

(c) What do you mean by stable and unstable equilibrium? Give examples. 3

(d) Discuss the importance of invariant interval in special theory of relativity. 3

(e) What are space-like, time-like intervals and light-like intervals? 3

(f) What is the meaning of critical velocity and turbulent motion? 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. The Lagrangian of an anharmonic oscillator is, $L(x,\dot{x}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - dx^3 + \beta x\dot{x}^2$. 6

3. Show that the motion of a particle under central force is planar. 6

4. A particle moving under a central force describes a spiral orbit given by $r = ae^{b\theta}$, where a, b are constants. Obtain the force law. 6

5. (a) What do you mean by light cone? Explain in 3-dimensional space. 3

(b) Explain longitudinal Doppler effect using 4-vector perspective. 3

6. Obtain the normal coordinates of a system of which the Lagrangian is given by 6

$L = \frac{1}{2}(m_1\dot{x}^2 + m_2\dot{y}^2) + \beta\dot{x}\dot{y} - \frac{1}{2}(x^2 + y^2)$. m_1, m_2 and β being constants.

7. Obtain the equation of continuity for a fluid flow. 6

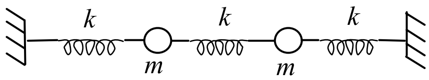
GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Explain the meaning of conjugation space. 2

(b) Show that symmetry in the Lagrangian leads to different constants of motion. 10

9. Two masses, each equal to *m* are connected by massless springs of spring constant *k*, such that they can freely slide on a smooth horizontal surface. The ends of the spring are fixed to vertical walls.



Determine:

(a) the normal frequencies. 4

(b) normal modes of vibration 4

(c) the normal coordinates. 4

10.(a) What do you mean by Minkowski space and define what are world lines? 4

(b) Explain the geometric interpretation of length contraction and time dilation using space time diagrams. 8

11.(a) A central attractive force varies as r^m. The velocity of a particle in a circular orbit of radius *r* is twice the escape velocity from the same radius. Find *m*. 4

(b) Show that ordinary 3-vector momentum is not conserved under Lorentz transformation whereas the 4-vector momentum is conserved under the Lorentz transformation. 8

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